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**Math-5630/6630**
Introduction to Numerical Analysis I
Summer 2007

Homework 7

## Programs

1. Use Taylor's polynomials to derive the formula:

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h} + O(h^4)\,.$$

## Programs

$$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{1}$$

$$f'(x) = \frac{f(x) - f(x-h)}{h} \tag{2}$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \tag{3}$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \tag{4}$$

1. Write a program that uses (3) and (4) to approximate $f'(1)$ and $f''(1)$ for $f(x) = e^x$ and $h = 1,\ 2^{-1},\ 2^{-2}, \ldots,\ 2^{-60}$ (if you can, also force single precision then you only need to go down to $2^{-30}$) . Format your output in columns as follows:

$h$ $f'$ error $f''$ error

Indicate the values of $h$ that give the least error.

2. Write a program that uses (3) and (4) to approximate $f'(1)$ and $f''(1)$ for $f(x) = e^x$ and $h = 1,\ 2^{-1},\ 2^{-2}, \ldots,\ 2^{-60}$, but this time use (one step of) Richardson's extrapolation (if you can, also force single precision then you only need to go down to $2^{-30}$). Format your output in columns as follows:

$h$ $f'$ error $f''$ error

Indicate the values of $h$ that give the least error, compare to previous computation.

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