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Math-5630/6630
Introduction to Numerical Analysis I
Summer 2007

Homework 6

Data: Consider the function

$$f(x) = \frac{1}{1 + 25x^2}$$

and nodes

$$x_0 = -3\,, \quad x_1 = -2\,, \quad x_2 = -1\,, \quad x_3 = -0.5\,, \quad x_4 = 0\,,$$

$$x_5 = 0.5\,, \quad x_6 = 1\,, \quad x_7 = 2\,, \quad \text{and} \quad x_8 = 3\,.$$

Programs
1. Write a program to compute the Lagrange interpolating polynomial. Compute the Lagrange interpolating polynomial of order 2 (for $x_0\,, x_4\,, x_8$), of order 4 (for $x_0\,, x_2\,, x_4\,, x_6\,, x_8$), of order 6 (for $x_0\,, x_1\,, x_2\,, x_4\,, x_6\,, x_7\,, x_8$), and of order 8 (for $x_0\,, x_1\,, x_2\,, x_3\,, x_4\,, x_5\,, x_6\,, x_7\,, x_8$). Graph these polynomials and evaluate them at $x = 0.75$.

2. Write a program to compute the interpolatory cubic spline (once you have set up the system of algebraic equations you may use Matlab built-in functions, e.g., mldivide, or \ "backslash" to solve the system). Compute the cubic spline with 2 subintervals (for $x_0\,, x_4\,, x_8$), with 4 subintervals (for $x_0\,, x_2\,, x_4\,, x_6\,, x_8$), with 6 subintervals (for $x_0\,, x_1\,, x_2\,, x_4\,, x_6\,, x_7\,, x_8$), and of with 8 subintervals (for $x_0\,, x_1\,, x_2\,, x_3\,, x_4\,, x_5\,, x_6\,, x_7\,, x_8$). Compute the spline of your choice (natural, clamped, smooth, or not-a-knot) and state which boundary conditions you used. Graph these splines and evaluate them at $x = 0.75$.

3. Compare the approximations you obtain in problem 1 and 2 to the exact function $f(x) = \frac{1}{1+25x^2}$, in particular evaluate $f(0.75)$.

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