A. J. Meir

Math-5630/6630
Introduction to Numerical Analysis I
Summer 2007

Homework 2

1. Use 2 digit rounding decimal arithmetic to compute the roots (using the quadratic formula) of the quadratic $x^2 + 10x + 4.75 = 0$. Compare to the exact roots and compute the absolute and relative errors.

2. Repeat the above but now compute the roots using the algorithm

$$x_1 = \frac{-b - \text{sign}(b)\sqrt{b^2 - 4ac}}{2a} \qquad \text{and} \qquad x_2 = \frac{c/a}{x_1}\,.$$

3. Show that $f(x) = x^3 - x - 1$ has exactly on root in the interval $[1, 2]$ (see problem 4).

*4. Suppose f is continuous on $[a, b]$ and that $f(a)f(b) < 0$ prove that f has at least one root in $[a, b]$. Assume that in addition f is differentiable on (a, b) and that the derivative is never 0, show that then the root is unique.

Program

1. Program the bisection algorithm (see Program 46, p. 252) with the following change: check that $f(a)f(b) < 0$, and stop with an error message otherwise.

a. Use your program to find an approximate root of $f(x) = x^3 - x - 1$ in the interval $[1, 2]$, which is within 10^{-6} of the exact root.
b. Use your program to find approximate roots of $f(x) = (2x^2 - 3x - 2)/(x - 1)$, which are within 10^{-6} of the exact roots. (there are two roots in $[-4, 4]$).

* Math 6630.